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ESTIMATING DEPTH-AVERAGED VELOCITIES IN ROUGH CHANNELS

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ABSTRACT

Profiles of streamwise velocity obtained from North Boulder Creek, Colorado, typically are non-logarithmic in form and exhibit the strong influence of form drag associated with coarse bed roughness. The spatially averaged profile is consistent with recent theoretical profile forms suggested for rough channels that are based on a partitioning of the total stress between a fluid part and a part associated with form drag on bed particles. Estimates of local depth-averaged velocity using algorithms that are based on several measurements in the flow column improve with explicit Riemann averaging, versus simple averaging, of the measurements. Estimates based on a single-point measurement at 0·6 of the flow depth, assuming a logarithmic or approximately logarithmic velocity profile, are the least reliable. Copyright © 2000 John Wiley & Sons, Ltd.

KEY WORDS: form drag; North Boulder Creek; Riemann average; velocity profile

INTRODUCTION

Local, vertical profiles of streamwise velocity in steep mountain streams are often irregular and non-logarithmic in form (e.g. Marchand *et al.*, 1984; Jarrett, 1990; Biron *et al.*, 1998), owing in part to bed roughness produced by coarse gravel and cobbles that extend into the water column a distance on the order of one-tenth of the flow depth or more. Previous attention to velocity profiles over coarse roughness has been given to obtaining effective values of the roughness height contained in the logarithmic velocity law for the purpose of estimating the depth-averaged velocity or computing flow resistance, assuming that a logarithmic profile is approximately valid over the entire flow depth (e.g. Bray, 1979; Griffiths, 1981; see also review by Pitlick, 1992). More recently, attention has been given to obtaining theoretical descriptions of the spatially averaged forms of velocity profiles over rough beds based on a partitioning of the total stress between a fluid part and a part associated with form drag on the bed particles (Nelson *et al.*, 1991; Wiberg and Smith, 1991). Herein we turn to a related practical issue that arises from the presence of irregular profiles in rough channels: how well the local depth-averaged velocity $\bar{u}$ can be estimated from only a few measurements in the flow column.

We describe a set of detailed velocity measurements from North Boulder Creek, Colorado, and briefly consider the forms of our measured velocity profiles in view of recent theoretical descriptions of spatially averaged profiles for rough channels (Nelson *et al.*, 1991; Wiberg and Smith, 1991). We then examine algorithms for using one to three measurements to estimate local depth-averaged velocities, including several that are based on the assumption of a logarithmic, or approximately logarithmic, velocity profile.

The motivation to examine simple algorithms for estimating local depth-averaged velocities arises from the practical need to efficiently obtain estimates of $\bar{u}$ at numerous channel position over a brief period. As one

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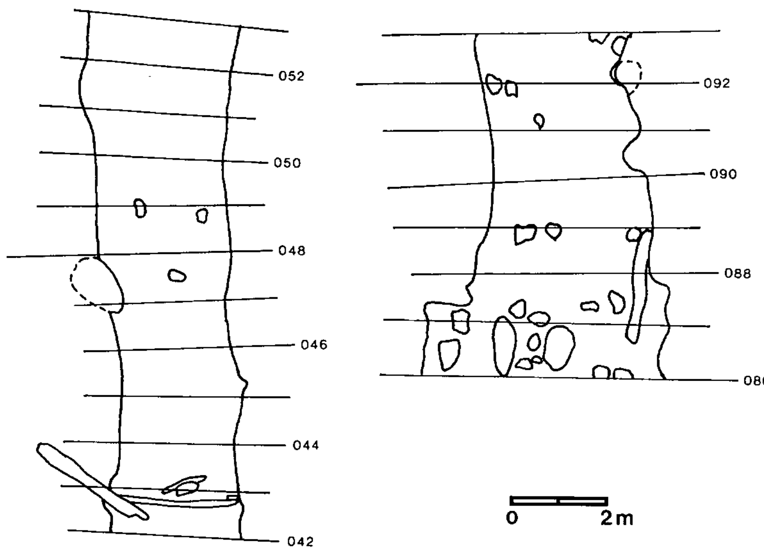

Figure 1. Maps of short reaches on North Boulder Creek showing section lines (044–051 and 088–091) for which velocity measurements were obtained at five locations equally spaced along each section (see text); section lines are spaced at 1 m intervals along channel centreline; mapped boulders include those larger than approximately 0·2 m; flow is from top to bottom

of many possible examples, our related field studies involve a program to systematically measure $\overline{u}$ at 120 cross-sections along a reach of North Boulder Creek. One objective has been to do this within a period of approximately constant flow discharge–a period of no more than three or four days in this case. For this reason, our measurement program has required striking a compromise between the number of positions at each of the 120 sections and the number of velocity measurements obtained at each position. Evaluating the accuracy(and error) of estimates of $\overline{u}$ based on one to three measurements at a position thus bears on the choice of numbers of positions versus velocity measurements in this type of program.

In the same vein, there is the practical issue of how two or three (or more) velocity measurements at a given position ought to be used in computing $\overline{u}$ – for example, whether $\overline{u}$ should be computed as a simple average or as some weighted average – in view of the highly variable forms of local velocity profiles. These profiles typically are non-logarithmic. Moreover, the theoretical treatments of Wiberg and Smith (1991) and Nelson *et al.* (1991) suggest that, over rough beds, the form of the profile varies with relative roughness (see description below). Together these points mean that no simple rule is available to estimate $\overline{u}$ based on one to three measurements, analogous to the '0·6 depth rule' associated with a logarithmic profile (Wiberg and Smith, 1991, p. 833). That is, for a fully logarithmic profile involving the flow depth h, the depth-averaged velocity occurs at a position h/e (*c.* $0{\cdot}37h$) from the bed; here it is assumed that $z_0 \ll h$, where z_0 is the nominal height above the bed at which the logarithmic profile goes to zero.

Measured values of the depth-averaged velocity $\overline{u}$ typically are used to calculate quantities such as total discharge (by the velocity–area method); but in addition, flow models that are based on depth-integrated forms of the continuity and momentum equations explicitly involve $\overline{u}$. Thus, an assessment of the accuracy of measured values of $\overline{u}$ is useful for field-based comparisons with such flow models.

FIELD MEASUREMENTS

The physical setting of the study reach on North Boulder Creek is described elsewhere (Furbish, 1987, 1993). The average bankfull width of the reach is about 2·9 m, the associated depth is about 0·4 m, and the average gradient is 0·034. Channel banks are steep, often nearly vertical. The bed consists of coarse gravel and cobbles; the average diameter is about 0·12 m (with variance of 0·0056 m ^{2}) based on a Wolman count of 204 particles on the bed of a large bend immediately downstream of the study reach. Judging from surveys of

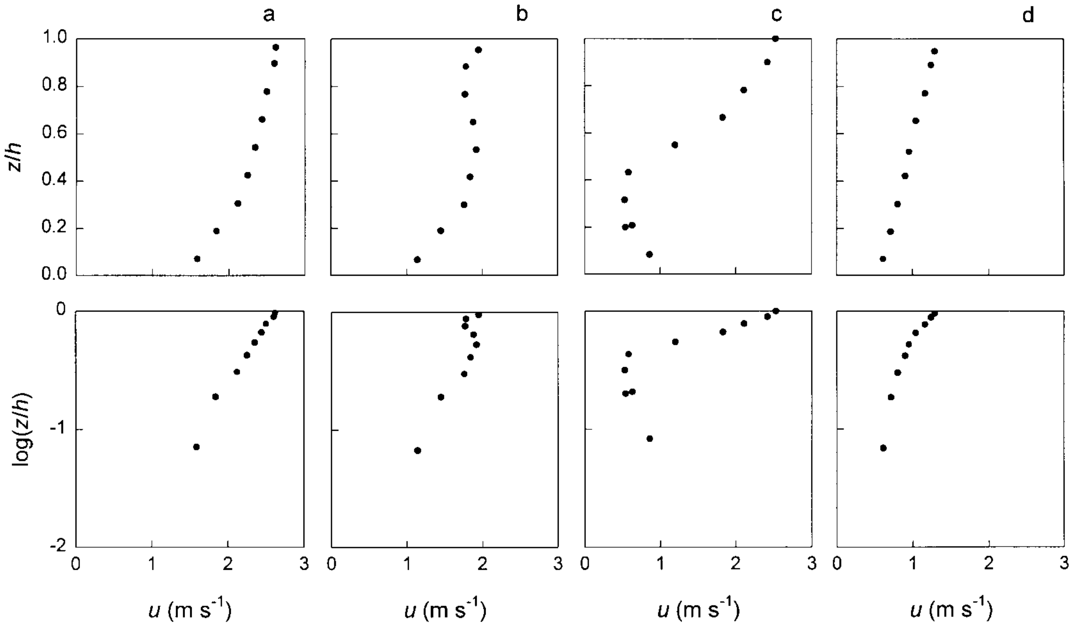

Figure 2. Examples of velocity profiles whose forms are: (a) approximately logarithmic; (b) S-shaped; (c) irregular: (d) approximately linear

transverse bed and water-surface profiles at 120 cross-sections equally spaced at 1 m intervals along the channel (see Figure 1; Furbish *et al.*, 1998, p. 3650), the ratio of roughness height to flow depth typically is on the order of 0·1 (or smaller) at bankfull stage. Nonetheless, large immobile boulders, areally covering 1 or 2 per cent of the total bed area, locally protrude far into (or above) the flow.

During June 1995, vertical profiles of time-averaged streamwise velocity at 60 positions were obtained. These positions were located systematically within two straight reaches (Figure 1) which, judging from previous surveys, had no unusually abrupt variations in bed topography (above the particle scale) nor large-amplitude bars. The 60 positions involved 12 cross-section lines spaced at 1 m (centreline) intervals; five positions at each section were located at the centreline and at ± 0·5 m and ± 1·0 m to the left and right of the centreline. The outer positions were at least 0·5 m away from the banks.

Each profile was based on measurements at eight to 10 approximately evenly spaced positions z in the water column, depending on local depth h, which varied from 0·22 m to 0·51 m. Inasmuch as possible, these positions were selected to be at common values of relative depth, z/h, to facilitate comparisons among profiles. Target values of z/h were 0·05, 0·17, 0·29, 0·41, 0·53, 0·64, 0·77 and 0·88, although actual values in some cases varied by a few per cent from these. In addition, we could sometimes add positions below $z/h = 0{\cdot}05$ or above $z/h = 0{\cdot}88$. In some cases, we could not get as low as $z/h = 0{\cdot}05$ with the meter. Each velocity measurement, obtained using a Teledyne Gurley (Pygmy) current meter, involved an averaging period of no less than 45 s. We opted to use this current meter because of the small vertical breadth (0·02 m) of its anemometer cups, and because of its ruggedness and reliability under field conditions that involved taking measurements during periods of intermittent rain, snow and hail, in air temperatures below 0° C. Related work (Cudney, 1995; Handel, 1996) has indicated that transverse velocity components at the site are small–typically more than an order of magnitude smaller than streamwise components–so that local velocities obtained with the meter are to a very good approximation equal to the streamwise components. Nonetheless, any inaccuracies produced by the presence of significant, local transverse velocity components are equally shared among different estimates of depth-averaged velocity $\bar{u}$ at each measurement position.

The 45 s (minimum) measurement period was adequate for obtaining a satisfactory time-averaged velocity, inasmuch as this period was longer than the characteristic time scale associated with large, coherent

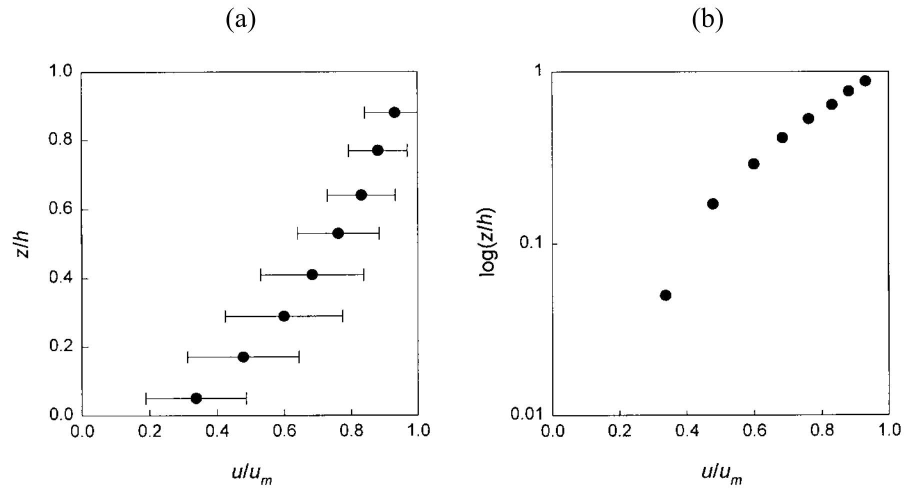

Figure 3. Plot of spatially averaged velocity profile obtained by averaging u/u_m at common values of (a) z/h with $\pm$ 1 standard deviation bars and (b) log z/h. Compare with figure 6 of Wiberg and Smith (1991) and figure 1 of Nelson *et al.* (1991)

turbulence fluctuations. For example, assuming that the horizontal length scale of large coherent flow structures ('eddies') is on the order of a channel width (c. 3 m), then with a reach-averaged streamwise velocity on the order of 1 m s^{-1}, about 15 wavelengths of such structures would be advected past a measurement site during 45 s. In this regard, we previously (June 1994) had measured time series of streamwise and transverse velocity components at section 044 (Figure 1). These series involved a sampling interval of 5 s over a period of more than 5 h using a two-component Marsh–McBirney electromagnetic current meter (model 511). The autocorrelation functions of the series decay very rapidly; the associated characteristic time scales are much less than 45 s.

ANALYSES AND RESULTS

About 10 per cent of the measured velocity profiles can be classified as being logarithmic, at least over the upper parts of the profiles, as indicated by straight-line fits to the data in semi-logarithmic plots involving u and z/h (Figure 2a). About 40 per cent of the profiles are approximately linear in form (Figure 2d). Of the remaining profiles, a few are S-shaped (Figure 2b) but most are irregular (Figure 2c), owing to the rough bed, and to local accelerations in the vicinity of large particles. Positions of maximum velocity u_m typically occur at or near the water surface, but sometimes these occur at a significant depth.

In contrast to the irregular velocity profiles that typically occur locally (Figure 2), the spatially averaged profile, computed by averaging values of u/u_m at common values of z/h for all 60 measurement positions (Figure 1), appears regular and smooth (Figure 3). Values of u/u_m were interpolated graphically between measured values when actual positions did not coincide with target positions of z/h. This averaged profile can be informally compared with the theoretical profiles of Wiberg and Smith (1991; see their figure 6) and Nelson *et al.* (1991; see their figure 1) obtained by an iterative algorithm that involves a partitioning of the total stress between a fluid part and a part associated with form drag on bed particles. Although this comparison does not constitute a test of the theories of Wiberg and Smith (1991) and Nelson *et al.* (1991), it is significant that our data conform well with their profiles for large relative roughness, including the characteristic decrease in the slope $\mathrm{d}(u/u_\mathrm{m})/\ \mathrm{d}[\log(z/h)]$ of the profile below $z/h \approx 0{\cdot}3$ in a plot involving $\log(z/h)$ (Figure 3b). (Also compare with data from Marchand *et al.* (1984) compiled in figures 3 and 4 of Wiberg and Smith (1991), and with figure 3 in Carling *et al.* (1998).

To examine simple algorithms for estimating local depth-averaged velocities, we first computed a value of $\overline{u}$ using all (eight to 10) measurements at each sampling position, and assumed that this represented a 'true'

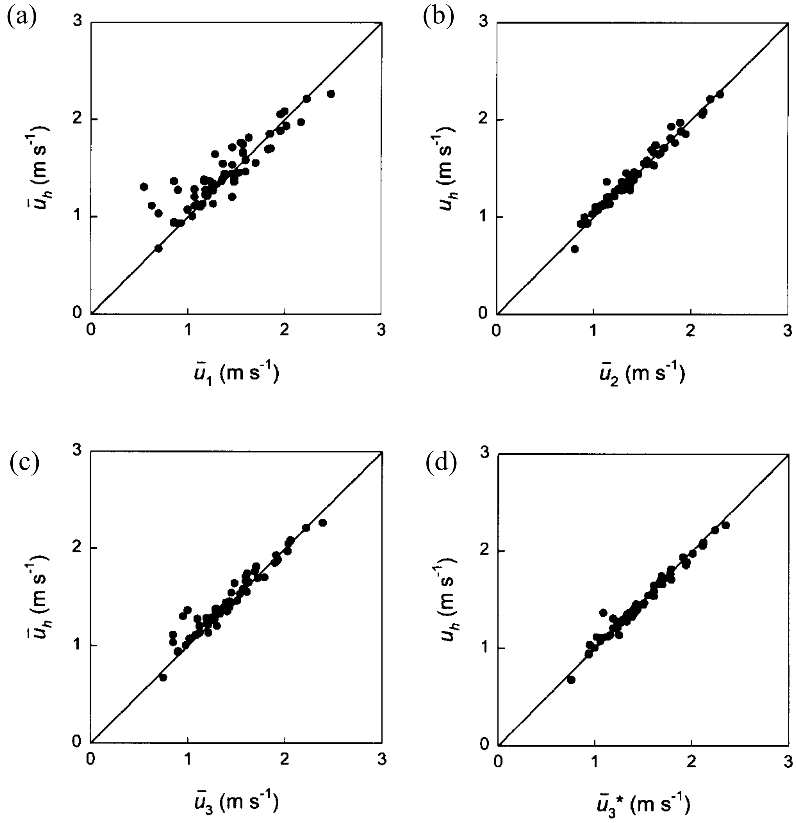

Figure 4. Plots of 'true' depth-averaged velocity $\overline{u}_h$ versus: (a) estimates of $\overline{u}_h$ based on one measurement at 0·4h ($\overline{u}_1$); (b) average of two measurements at 0·2h and 0·8h ($\overline{u}_2$); (c) 'double average' of three measurements at 0·2h, 0·4h and 0·8h ($\overline{u}_3$); (d) Riemann-weighted values at 0·2h, 0·6h and $0 \cdot 8h(\overline{u}_3^*\backslash$

value, denoted by $\overline{u}_h$. Each value of $\overline{u}_h$ was obtained using a Riemann summation (e.g. Swokowski, 1975, p. 185) with interval limits located at midpoints between adjacent data positions. (This choice of interval limits was a matter of convenience, and coincides with datum-centred intervals over most of the flow depth.) We then recomputed $\overline{u}$ using one to three of our measured values of $u(z)$ at each sampling position. Various algorithms recommended by the US Geological Survey (1977) include: obtaining one measurement at 0·4h (that is, at 0·6 of the depth); obtaining two measurements at 0·2h and 0·8h; and obtaining three measurements at 0·2h, 0·4h and 0·8h. Then, because the numerical weighting attached to each velocity measurement is programmed into the selection of the depths assuming a logarithmic (or approximately logarithmic) profile, estimates of $\overline{u}$, respectively denoted by $\overline{u}_1$, $\overline{u}_2$ and $\overline{u}_3$, involve directly using the one measurement in the case of $\overline{u}_1$, simple averaging in the case of $\overline{u}_2$, and 'double averaging' in the case of $\overline{u}_3$ (where the average of the measurements at 0·2h and 0·8h is averaged with the measurement at 0·4h).

A plot of $\overline{u}_h$ versus $\overline{u}_1$ (Figure 4a) suggests that $\overline{u}_1$, on average, underestimates $\overline{u}_h$. This error increases with decreasing $\overline{u}_h$. Judging from the spatially averaged profile (Figure 3a), the position of the velocity $u(z)$ that coincides with $\overline{u}_h$ is (on average) higher in the water column than 0·4h (0·6 of the depth). This is consistent with points of Wiberg and Smith (1991, p. 833) in reference to the measurements of Bathurst (1988), that the difference between mean velocities and velocities at 0·4h, based both on measured profiles and on profiles obtained from stress-partition calculations, was about 10 per cent (decreasing with relative roughness). Our measurements suggest that $\overline{u}_1$, on average, underestimates $\overline{u}_h$ by about 6 per cent. Similar conclusions pertain to the estimates $\overline{u}_2$ and $\overline{u}_3$ (Figure 4b and c); on average, $\overline{u}_2$ and $\overline{u}_3$ respectively underestimate $\overline{u}_h$ by about 1 and 3 per cent.

Using a simple Riemann average of measurements at 0·2*h*, 0·4*h* and 0·8*h* (with interval limits at 0·3*h* and 0·6*h*), estimates of $\overline{u}$ based on these three measurements (denoted by $\overline{u}_3^*$) systematically improve (Figure 4d). On average, $\overline{u}_3^*$overestimates $\overline{u}_h$ by about 0·1 per cent. Nonetheless, a residual variance, related to deviations in the forms of local profiles from the reach-averaged form, is associated with such estimates. Standard errors for $\overline{u}_1$, $\overline{u}_2$, $\overline{u}_3$ and $\overline{u}_3^*$about the lines of perfect fit in Figure 4 are respectively 0·14, 0·056, 0·079 and 0·042 (each with units of metres per second).

Judging from these standard errors, the estimate $\overline{u}_2$, which is based on two measurement positions, is better than the estimate $\overline{u}_3$, which is based on three; moreover, $\overline{u}_2$ is nearly as good an estimate of $\overline{u}_h$ as $\overline{u}_3^*$(Figure 4). This point is also consistent with the conclusions of Wiberg and Smith (1991, p. 833), that 'mean velocity estimates computed from the average of the velocities at 0·2*h* and 0·8*h* in [their] calculated profiles differ from mean velocities computed from the whole profile by less than 3% even for large values of relative roughness'. Because a velocity measured at 0·4*h*, on average, underestimates the depth-averaged velocity $\overline{u}$, an estimate of $\overline{u}$ based solely on a velocity measured at 0·4*h* (that is, estimate $\overline{u}_1$), or which involves a simple arithmetic average of a velocity measured at 0·4*h* (that is, estimate $\overline{u}_3$), generally underestimates $\overline{u}_h$ because the averaging overly weights the significance of the velocity at this position. In contrast, a simple average of measurements obtained at 0·2*h* and 0·8*h* (that is, estimate $\overline{u}_2$) provides a better estimate of $\overline{u}$. Qualitatively, the velocities at 0·2*h* and 0·8*h* are close to the velocities at 0·25*h* and 0·75*h* in the spatially averaged profile (Figure 3a). In this situation, the simple average of the velocities at 0·2*h* and 0·8*h*, which is equivalent to a Riemann average involving two intervals separated at $z/h = 0{\cdot}5$, is very close to the 'exact' Riemann average involving the velocities at 0·25*h* and 0·75*h*, insofar as the spatially averaged profile (Figure 3a) is well approximated by two straight-line segments above and below $z/h \approx 0{\cdot}5$. Moreover, $\overline{u}_2$ provides an exact estimate of $\overline{u}$ in the limiting case of a linear profile, which represents approximately 40 percent of our profiles.

CONCLUSIONS

Our measured profiles of streamwise velocity collectively exhibit the influence of form drag associated with coarse bed roughness, in addition to local accelerations associated with individual large particles, and perhaps small bed forms. The spatially averaged profile qualitatively conforms well with the theoretical profiles of Wiberg and Smith (1991) and Nelson *et al.* (1991) for large relative roughness, including the characteristic decrease in the slope $d(u/u_m)/d[\log(z/h)]$ of the profile below $z/h \approx 0{\cdot}3$ in a plot involving $\log(z/h)$ versus u/u_m (Figure 3b).

The theoretical profiles of Wiberg and Smith (1991) and Nelson *et al.* (1991), however, do not possess a self-similarity over *z/h* for varying values of relative roughness in the sense that a logarithmic profile does (Wiberg and Smith, 1991, p. 833). This means that no simple rule is available to estimate $\overline{u}$ based on one to three measurements, analogous to the '0·6 depth rule' associated with a logarithmic profile. In view of this, our results suggest a straightforward approach: in the absence of detailed profiles of *u(z)* over large relative roughness, one would probably do well to choose a simple Riemann average of two or three measurements to estimate $\overline{u}$ rather than adopting a procedure that is based on the assumption of a logarithmic profile. Our data confirm the point anticipated by Wiberg and Smith (1991), that the simple average of measurements at 0·2*h* and 0·8*h*, which is equivalent to a Riemann average involving two intervals separated at $z/h = 0{\cdot}5$, is a reasonably robust estimate of $\overline{u}$ (Figure 4b) given that profiles with increasing relative roughness become less logarithmic and more linear in form. Using this algorithm, the standard error of estimates of $\overline{u}$ for our data set is 0·056 m s $^{-1}$. A Riemann average involving three measurements at 0·2*h*, 0·4*h* and 0·8*h* further reduces this standard error to 0·042 m s $^{-1}$ (Figure 4d). A useful property of this algorithm is that its uneven spacing of positions concentrates the lower two in that part of the flow column where the velocity gradient is most likely to be steep; and its three positions are well suited for estimating $\overline{u}$ in the case of a logarithmic profile. Whereas the error of the estimate of $\overline{u}$ using this Riemann average may be acceptably small in some situations, improving such estimates requires obtaining more than three measurements at a site.

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